

Problem set 4

Problems and exercise numbers are from CLRS 2nd edition.

1. (C.2-2) Professor Rosencrantz flips a fair coin once. Professor Guildenstern flips a fair coin twice. What is the probability that Professor Rosencrantz obtains more heads than Professor Guildenstern?
2. (C.2-3) A deck of 10 cards, each bearing a distinct number from 1 to 10, is shuffled to mix the cards thoroughly. Three cards are removed one at a time from the deck. What is the probability that the three cards are selected in sorted (increasing) order?
3. (C.2-9*) You are a contestant in a game show in which a prize is hidden behind one of three curtains. You will win the prize if you select the correct curtain. After you have picked one curtain but before the curtain is lifted, the emcee lifts one of the other curtains, knowing that it will reveal an empty stage, and asks if you would like to switch from your current selection to the remaining curtain. How would your chances change if you switch?
4. (5.3-2) Professor Kelp decides to write a procedure that will produce at random any permutation besides the identity permutation. He proposes the following procedure:

```
PERMUTE-WITHOUT-IDENTITY(A)
1  n ← length[A]
2  for i ← 1 to n − 1
3      swap A[i] ↔ A[RANDOM(i + 1, n)]
```

Does this code do what professor Kelp intends?

5. (5.3-3) Suppose that instead of swapping element $A[i]$ with a random element from the subarray $A[i \dots n]$, we swapped it with a random element from anywhere in the array:

```
PERMUTE-WITH-ALL(A)
1  n ← length[A]
2  for i ← 1 to n
3      swap A[i] ↔ A[RANDOM(1, n)]
```

Does this code produce a uniformly random permutation? Why or why not?

6. (8-4 a,c) Water jugs. Suppose that you are given n red and n blue water jugs, all of different shapes and sizes. All red jugs hold different amounts of water, as do the blue ones. Moreover, for every red jug, there is a blue jug that holds the same amount of water, and vice versa.

It is your task to find a grouping of the jugs into pairs of red and blue jugs that hold the same amount of water. To do so, you may perform the following operation: pick a pair of jugs in which one is red and one is blue, fill the red jug with water, and then pour the water into the blue jug. This operation will tell you whether the red or the blue jug can hold more water, or if they are of the same volume. Assume that such a comparison takes one time unit. Your goal is to find an algorithm that makes a minimum number of comparisons to determine the grouping. Remember that you may not directly compare two red jugs or two blue jugs.

- a. Describe a deterministic algorithm that uses $O(n^2)$ comparisons to group the jugs into pairs.
 - c. Give a randomized algorithm whose expected number of comparisons is $O(n \log n)$, and prove that this bound is correct. What is the worst-case number of comparisons for your algorithm?
7. You have to sort an array of length n . Assume that not all permutations are equally likely to be seen in the input: with probability $1/3$ the input comes already sorted, with probability $1/3$ the input is sorted in the opposite way, and any other permutation has the same probability to be in the input. Knowing that, you use the following algorithm for sorting: First check whether the input is sorted or whether it is sorted in the opposite way. If not, use quicksort. What is the overall expected running time of such an algorithm?