

## Problem set 3

Problem and exercise numbers are from CLRS 2nd edition.

- For each of the following problems either give a polynomial-time algorithm or prove that the problem is NP-complete:

1-SAT: Given: A set of variables  $X = \{x_1 \dots x_n\}$  and a set  $C = \{c_1 \dots c_m\}$  of clauses where each clause is a single literal. Question: does there exist a truth assignment for  $X$  such that each clause has a true literal?

5-SAT: Given: A set of variables  $X = \{x_1 \dots x_n\}$  and a set  $C = \{c_1 \dots c_m\}$  of clauses where each clause is a collection of 5 literals. Question: does there exist a truth assignment for  $X$  such that each clause has a true literal?

- 34.1-5: Show that an otherwise polynomial-time algorithm that makes at most a constant number of calls to polynomial-time subroutines runs in polynomial time, but that a polynomial number of calls to polynomial-time subroutines may result in an exponential-time algorithm.
- 34.5-8: In the half 3-CNF satisfiability problem, we are given a 3-CNF formula  $\phi$  with  $n$  variables and  $m$  clauses, where  $m$  is even. We wish to determine whether there exists a truth assignment to the variables of  $\phi$  such that exactly half the clauses evaluate to 0 and exactly half the clauses evaluate to 1. Prove that the half 3-CNF satisfiability problem is NP-complete.
- 34-1: Independent set. An independent set of a graph  $G = (V, E)$  is a subset  $V' \subseteq V$  of vertices such that each edge in  $E$  is incident on at most one vertex in  $V'$ . The independent-set problem is to find a maximum-size independent set in  $G$ . Formulate a related decision problem for the independent-set problem, and prove that it is NP-complete. (Hint: Reduce from the clique problem.)
- Show that CLIQUE is NP-complete by a reduction from VERTEX-COVER. Show that VERTEX-COVER is NP-complete by a reduction from CLIQUE.
- 34.2-2: Prove that if  $G$  is an undirected bipartite graph with an odd number of vertices, then  $G$  is nonhamiltonian.
- 34.2-11: Let  $G$  be a connected, undirected graph with at least 3 vertices, and let  $G^3$  be the graph obtained by connecting all pairs of vertices that are connected by a path in  $G$  of length at most 3. Prove that  $G^3$  is hamiltonian. (Hint: Construct a spanning tree for  $G$ , and use an inductive argument.)
- 34.5-6: Show that the hamiltonian-path problem is NP-complete.
- 17.1-1: If the set of stack operations included a MULTIPUSH operation, which pushes  $k$  items onto the stack, would the  $O(1)$  bound on the amortized cost of stack operations continue to hold?
- 17.3-4: What is the total cost of executing  $n$  of the stack operations PUSH, POP, and MULTIPOP, assuming that the stack begins with  $s_0$  objects and finishes with  $s_n$  objects?

Extra credit problem: Is there a **simple** proof of NP-completeness of the Hamiltonian cycle problem?