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Overlay and P2P Networks

On power law networks

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On Zipf's distribution and power-laws

A power-law implies that small occurrences are extremely common, whereas large instances are extremely rare.

This regularity or law is also referred to as Zipf or Pareto

Zipf is used to model the **rank** distributions, and power-law for **frequency** distributions

The linguist George Zipf first proposed the law in 1935 in the context of word frequencies in languages.

For Web sites, the Zipf law means that large sites get disproportionately more traffic than smaller sites.

Zipf's Law

$F \sim R^{-\beta}$, where R is the rank and the constant is close to one

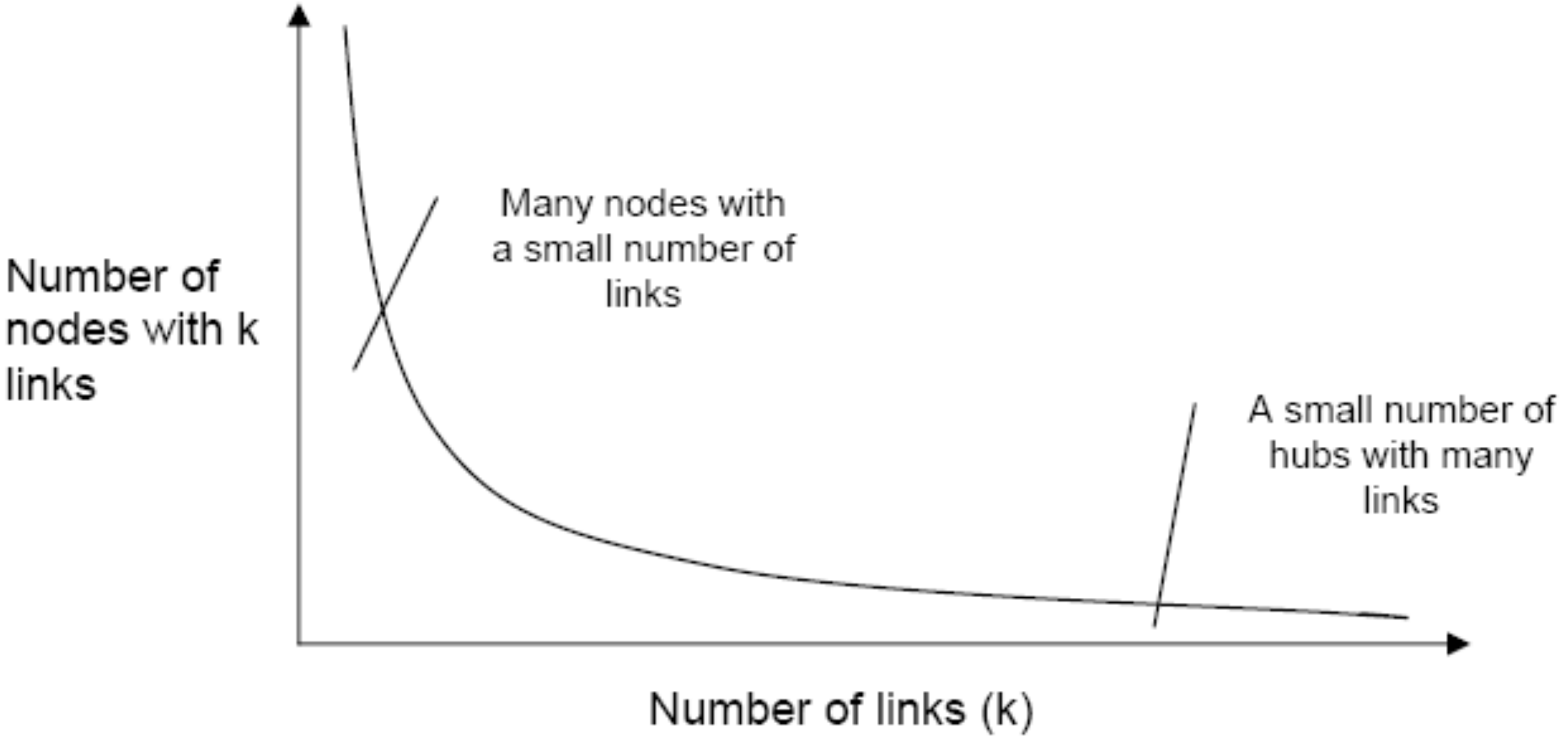
Straight line on a log-log plot

Zipf's law has been used to model Web links and media file references. It has therefore profound implications for content delivery on the Internet.

Efficient caching relies heavily on Zipf's law to replicate a small number of immensely popular files near the users.

The distribution of the number of connections a host has to other hosts on the Internet has been shown to follow the Zipf distribution.

Power-law distribution



Observations

Gnutella (v0.7) and Freenet support the formation of hubs

They are power law networks

How robust are these networks?

Robustness

Given a certain expected network structure, a very interesting question is how easy it is to disrupt the network and partition it into disjoint parts

Cohen et al. have analytically shown that networks in which the vertex connectivity follows a power-law distribution with an index of at most (alpha<3) are very robust in the face of random node breakdowns.

$$p \leq 1 + \left(1 - m^{\alpha-2} K^{3-\alpha} \frac{\alpha - 2}{3 - \alpha}\right)^{-1},$$

Where p is a bound on network partitioning, m is the minimum node degree, and K is the maximum node degree.

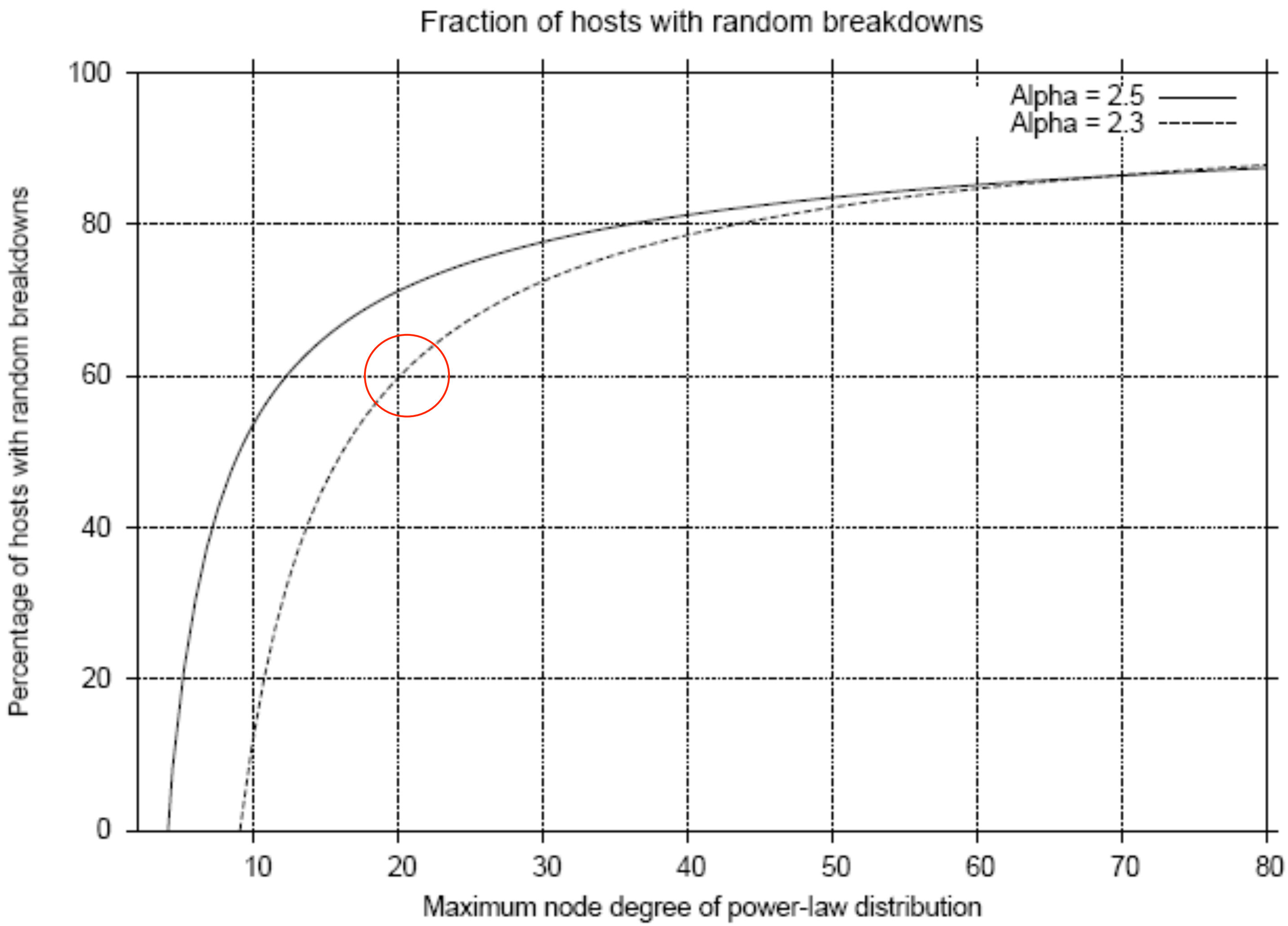
Robustness II

The Internet node connectivity has been shown to follow a power-law distribution with alpha=2.5

Similar investigation has been made for the Gnutella P2P network resulting in the observation that alpha = 2.3

Both the Internet and Gnutella present a highly robust topology. They are able to tolerate random node breakdowns

Resiliency of power-law networks



Gnutella Robustness

For a maximum and fairly typical node degree of 20, the Gnutella overlay is partitioned into disjoint parts only when more than 60% of the nodes are down

Robustness is a highly desirable property in a network. The above equation is useful in understanding the robustness of power-law networks; however, it assumes that the node failures are random

Although a power-law network tolerates random node failures well, it is still vulnerable to selective attacks against nodes

An orchestrated attack against hubs in the network may be very effective in partitioning the network

Small Worlds: Milgram’s experiment

The Small-World Problem – Milgram (1967)

How many intermediaries are needed to move a letter from person A to person B through a chain of acquaintances?

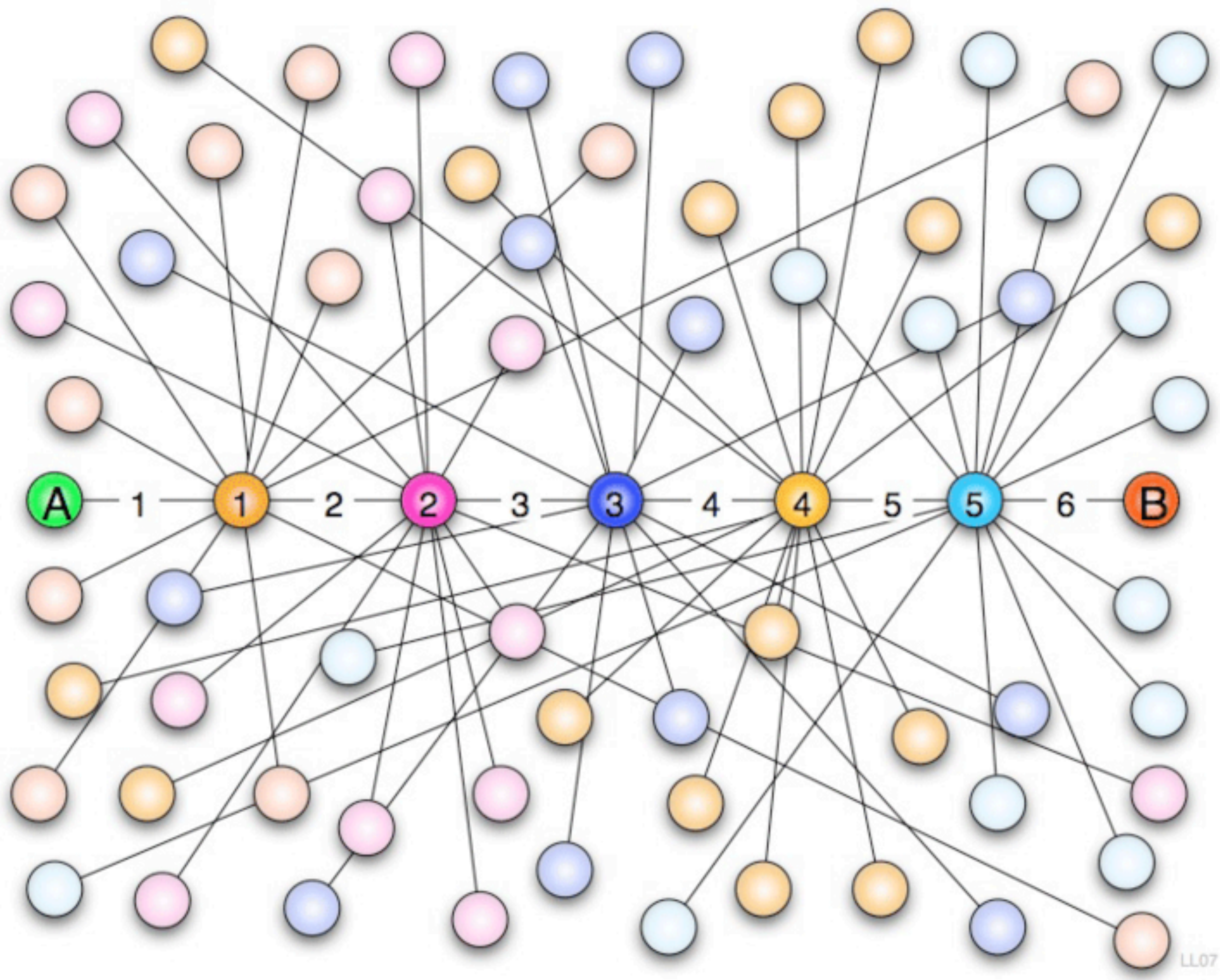
Designed to find out average path length

Letter-sending experiment: starting in Nebraska/Kansas,with a target person in Boston.

People forwarded the message towards the target person.

Six degrees of separation.

Six degrees of separation



Source: Wikipedia

Small Worlds

Small-world networks are characterized by a **graph degree power-law distribution**

They exhibit **clustering** and thus are different from random networks (preferential attachment)

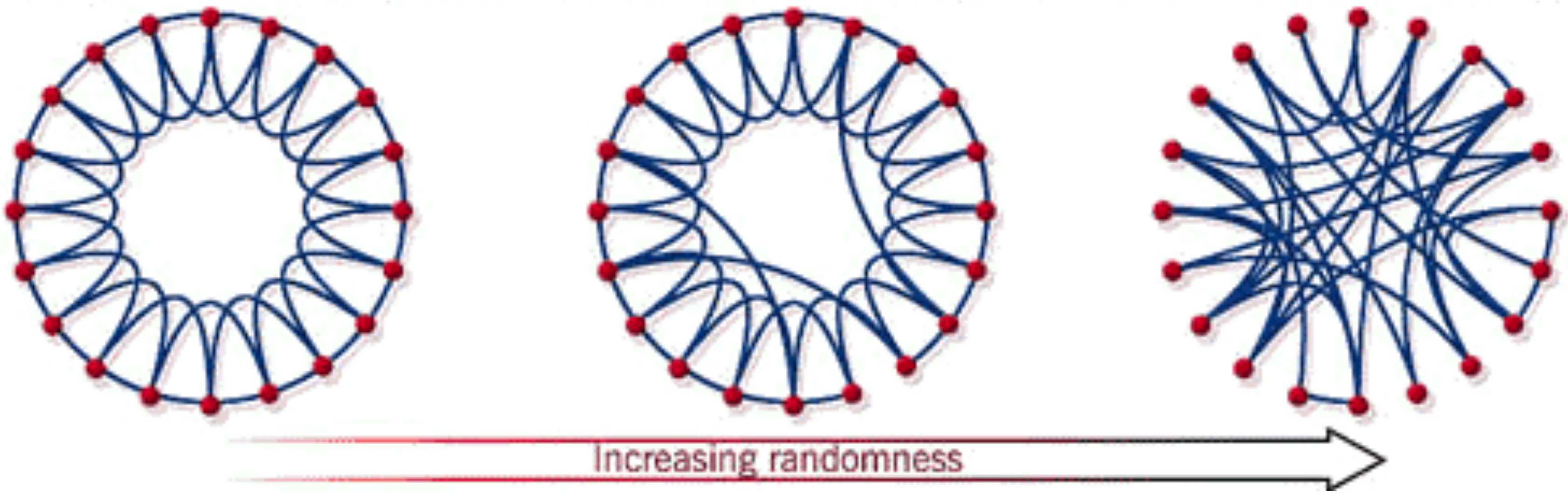
Most nodes have relatively few local connections to other nodes, but a significant small number of nodes have large wide-ranging sets of connections

The small-world topology enables efficient short paths because the well connected nodes provide shortcuts across the network

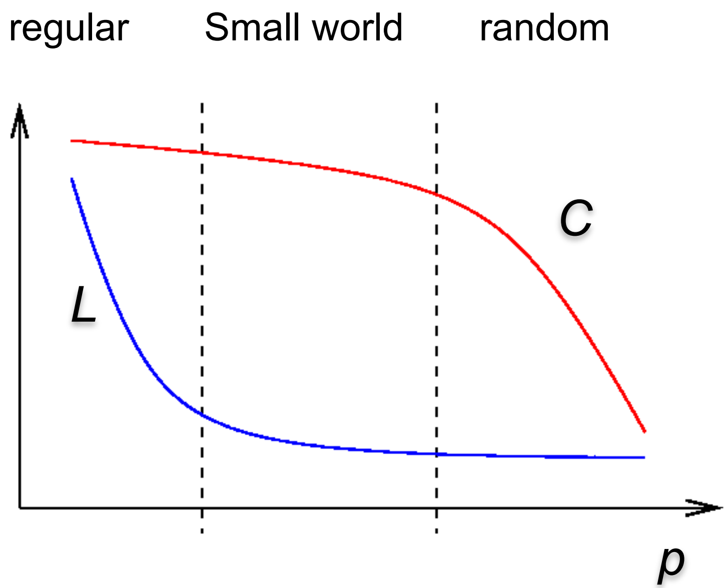
Structured network
- high *clustering*
- large diameter
- regular

Small-world network
- high *clustering*
- small diameter
- almost regular

Random network
- small *clustering*
- small diameter



Reference: Duncan J. Watts & Steven H. Strogatz, Nature 393, 440-442 (1998)



C(p) : clustering coefficient
L(p) : average path length

Reference: Duncan J. Watts & Steven H. Strogatz, Nature 393, 440-442 (1998)

Kleinberg's result I/III

Jon Kleinberg showed that it is possible to do efficient routing on grids with the small world property

The possibility of efficient routing depends on a balance between the proportion of shortcut edges of different lengths with respect to coordinates in the base grid

The key idea is to use a frequency of edges of different lengths that decrease **inverse proportionally** to the length

Kleinberg’s result II/III

Results in an infinite family of small world network models on a grid with power-law distributed random long-range links

K(n,k,p,q,r)
p – radius of neighbours to which short local links
q – number of random long range links
k - dimension of the mesh
r - clustering exponent of inverse power-law distribution.

Prob.[(x,y)] $\propto$ dist(x,y)$^{-r}$

Expected Delivery time =

$O((\log n)^2)$, for r = 2 (and the special case k=r).
$\Omega(n^{(2-r)/3})$, for $0 \leq r < 2$.
$\Omega(n^{(r-2)/(r-1)})$, for $2 < r$.

Kleinberg’s result III

Simple greedy routing can find routes in $O(\log^2(n))$ where n is the size of the graph

- Decentralized
- Decisions based on local information

Later work has investigated other topologies than grids (rings, …) and improving efficiency through topology information, cues, etc.

Applications of Small World Networks

Many applications in peer-to-peer networks

The Gnutella network has been observed to exhibit the clustering and short path lengths of a small world network. Its overlay dynamics lead to a biased connectivity among peers where each peer is more likely connected to peers with higher uptime

The Freenet routing algorithm is built on the small world assumption

Other applications in distributed hashing (DHTs) such as Symphony that uses long-range contacts drawn randomly from a family of harmonic distributions

Is Freenet a small world?

There must be a scale-free power-law distribution of links within the network.

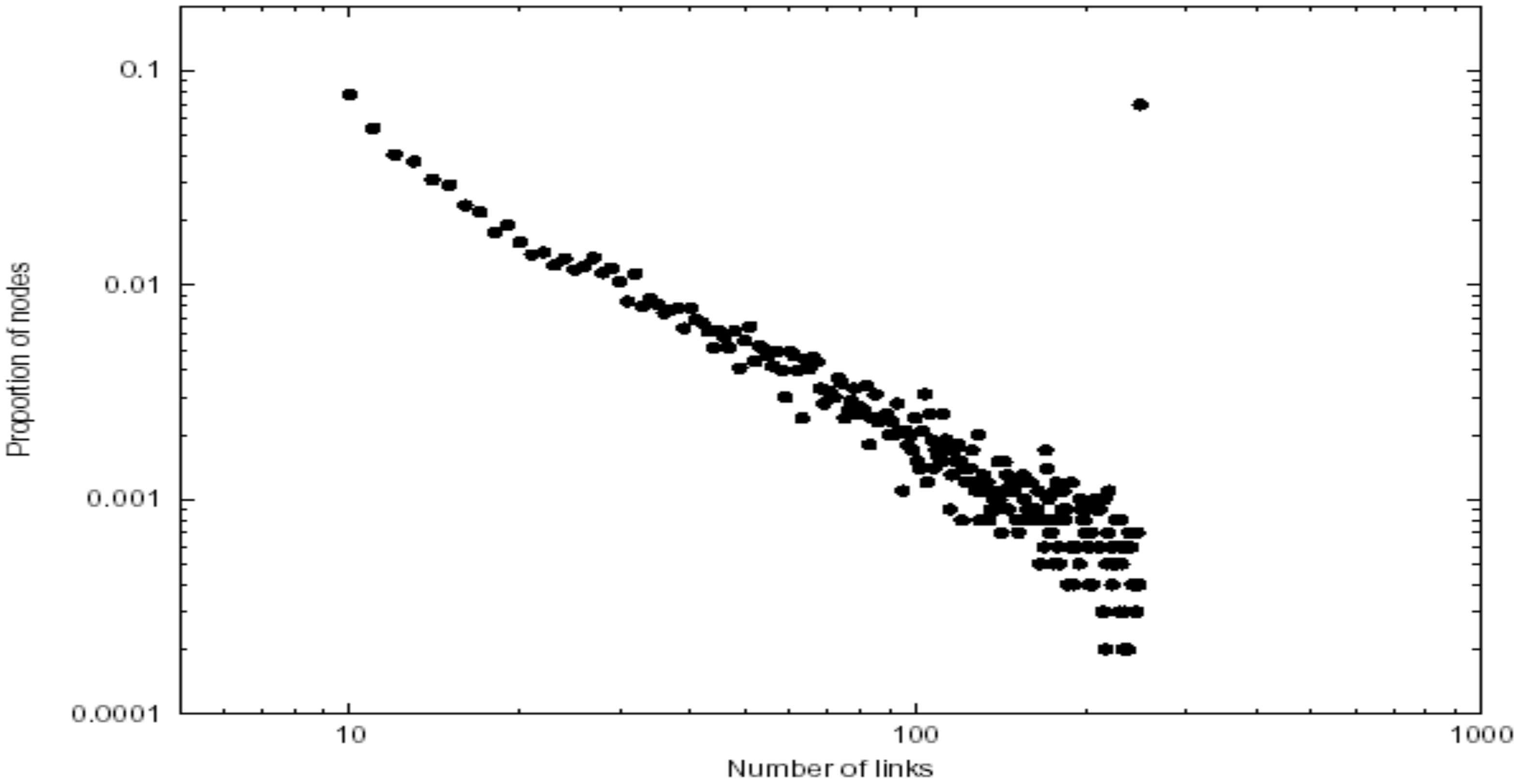


Fig. 5. Distribution of link number among Freenet nodes.

Source: www.ics.forth.gr/dcs/Activities/Projects/p2p/ploumid-freenet.ppt