582206 Models of Computation (Autumn 2010)
2nd course exam 16 December, 16–19 o'clock, Exactum A111
examiner Jyrki Kivinen

To speed up grading, please give your answers to problems 1, 2 and 3 each on their own sheet.
Solve all parts of all problems. The maximum score is 24 points.

1. [*2+3+3 points*]

   (a) Give a context-free grammar for the language $\{ 0^m 1^n 0^k 1^m \mid m, n, k \in \mathbb{N} \}$ over the alphabet $\{ 0, 1 \}$.

   (b) Give a context-free grammar for the language consisting of all the regular expressions over the alphabet $\{ 0, 1 \}$. Thus, the language you generate is over the alphabet $\{ 0, 1, (,), \circ, \cup, ^* \}$ and contains, for example, strings $0 \cup 1$ and $(0 \circ 1 \cup 1^*)^*$, but not $1\cup$ or $1 \circ \cup 0$.

   (c) Using the method covered in the course, convert the grammar

   $$\begin{aligned} S &\rightarrow \mathrm{a}S\mathrm{c} \mid T \\ T &\rightarrow \mathrm{b}T \mid \varepsilon \end{aligned}$$

   into an equivalent push-down automaton. For a solution it is sufficient to show the state diagram of the automaton. Give also a left-most derivation of the string aabcc in the grammar, and simulate an accepting computation of your automaton with that input (by for example listing the intermediate configurations of the stack).

2. [*3+2+2 points*]

   (a) A *dominating set* in a graph $G = (V, E)$ is a set of vertices that covers at least one end point of each edge. Hence, $U \subseteq V$ is a dominating set if for all $(u, v) \in E$ we have $u \in U$ or $v \in U$. Consider the formal language

   $$DS = \{ \langle G, k \rangle \mid \text{graph } G \text{ has a dominating set with exactly } k \text{ vertices} \} .$$

   Show that $DS \in \mathrm{NP}$. Give the proof on a fairly high level without getting too deep into technical details of Turing machines etc.

   (b) Actually we know that the language DS from part (a) is NP-complete. What implications does this have for possible practical algorithms for solving the problem?

   (c) Give an example of a formal language that is recognizable but not decidable. What implications do these properties have for possible practical algorithms for solving your problem?

3. [*3+3+3 points*]

   (a) The language $\{ 0^n 1^n 0^n 1^n \mid n \in \mathbb{N} \}$ is known not to be context-free. Using this fact prove that the class of context-free languages is not closed under intersection.

   (b) Use the result from part (a) together with other known facts about context-free languages to prove that the class of context-free languages is not closed under complementation.

   (c) Prove that the language $D = \{ \langle M \rangle \mid \langle M \rangle \notin L(M) \}$ is not Turing-recognizable.

Tehtävät suomeksi kääntöpuolella!