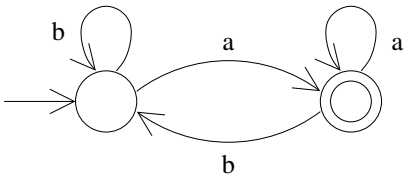
582206 Models of Computation (Autumn 2010)
Course exam #1 (21 October)
examiner: Jyrki Kivinen

To speed up grading, please give your answers to each of the problems 1, 2, 3 and 4 on its own sheet.
When you are asked to give an automaton, just draw a state diagram, no more formal definition is needed.
Answer all parts of all questions. The maximum for this exam is 24 points.

1. [*2+2+2 points*] Give both a finite automaton (deterministic or nondeterministic, whichever you prefer) and a regular expression for the following languages over the alphabet $\{ \mathrm{a}, \mathrm{b} \}$:
   (a) strings in which the first two symbols are either "ab" or "ba"
   (b) strings ending with "bab"
   (c) string in which the difference between the number of 'a' symbols and the number of 'b' symbols is evenly divisible by 3.

2. [*1+5 points*]
   (a) Describe verbally the language recognised by the automaton below. Also give for the language as simple a regular expression as possible.



   (b) Use the procedure described in the text book to convert the automaton from part (a) into a regular expression. If you show the intermediate results clearly, no other explanations are needed.

3. [*6 points*] Show that the language

$$A = \{ \mathrm{a}^m \mathrm{b}^p \mathrm{a}^n \mathrm{b}^q \mid m + n = p + q \}$$

   over the alphabet $\{ \mathrm{a}, \mathrm{b} \}$ is not regular. You may use the pumping lemma and any other general properties of regular languages you know from the course, but not results stating directly that some specific language is not regular.

4. [*2+4 points*] Recall the definition of the reverse of a string: if $w = w_1 \dots w_n$ where $w_i \in \Sigma$ for all i, then $w^{\mathcal{R}} = w_n \dots w_1$. Further, for a language $A \subseteq \Sigma^*$ we use the notation $A^{\mathcal{R}} = \{ w^{\mathcal{R}} \mid w \in A \}$.
   We say that a language A is *symmetrical*, if $A^{\mathcal{R}} = A$.
   (a) Are all symmetrical languages regular? Are all regular languages symmetrical?
   (b) Is the class of symmetrical languages closed under the union operation? How about the concatenation operation?

   Justify your answers clearly and precisely (but not necessarily at great length).